Bayesian Workflow

Maximizing the odds of building a useful model!

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Bayesian Workflow

Scope out your problem
What inputs and outputs can help you learn? What relationships can you see by eye?

Specify likelihood & priors
Use knowledge of the problem to construct a generative model and shape the scope of the parameters

Check the model with fake data
Generate data, fit model, and evaluate fits (SBC)

Fit the model to real data
To recover parameters

Check diagnostics
Algorithms should come with diagnostics that let you know when they're not working

Graph fit estimates
Understand your inferences

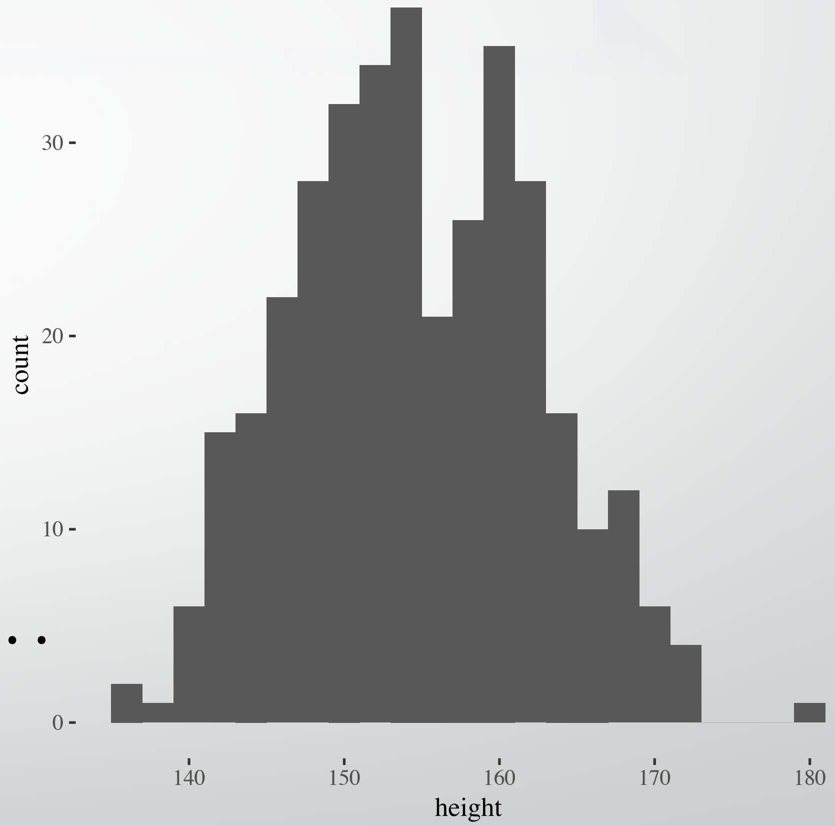
Check predictive posterior
Perform PPCs to understand predictions

Compare models
Iterate on model design, choose a model

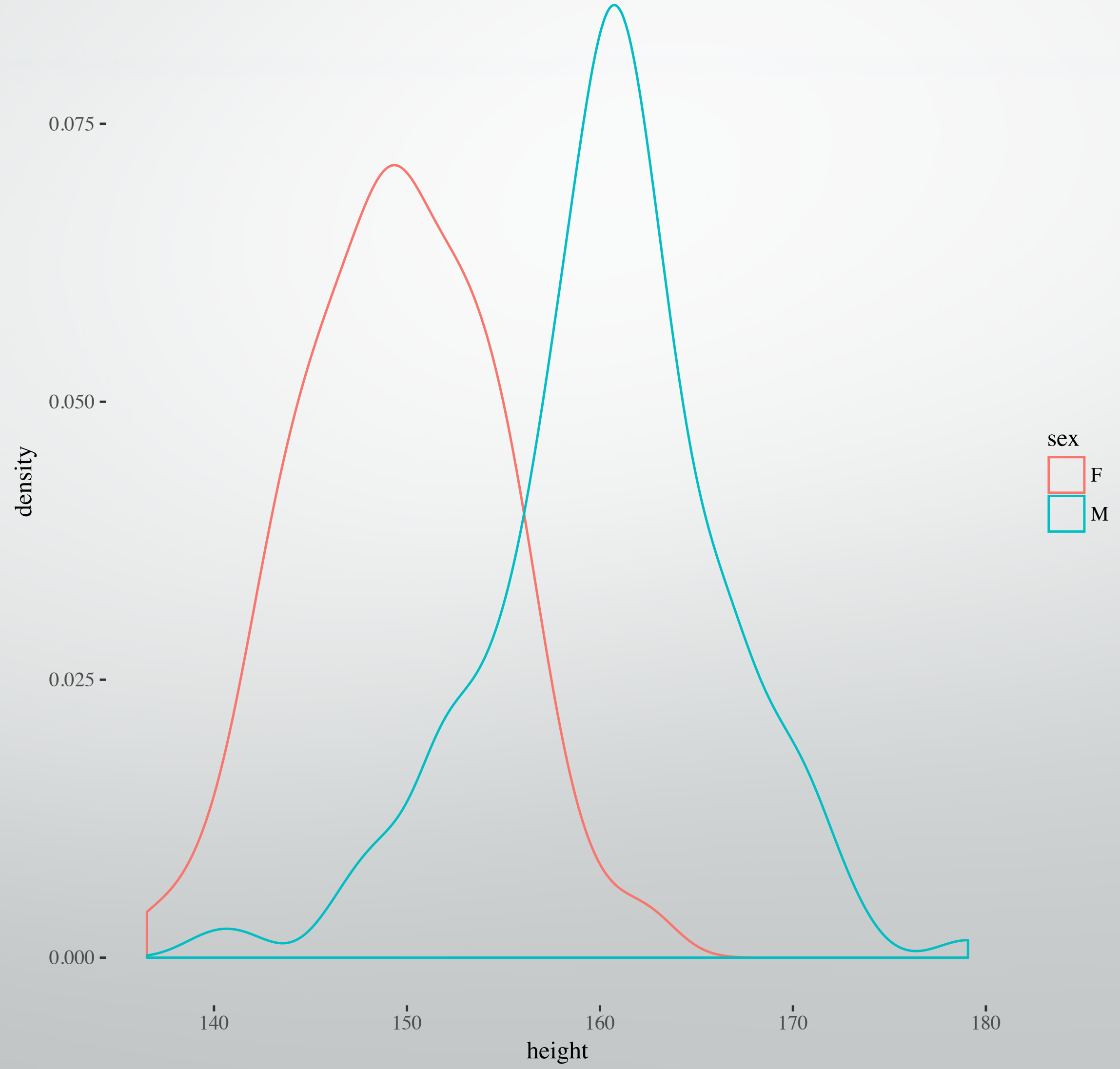
Problem | Model | Fake Data | Fit | Diagnostics | Graph fit | PPCs | Compare

- Height data collected in the 1960s on the !Kung San foraging population
- We want to better understand the population
- Explore how the measurements we have are related to each other

```
'data.frame': 352 obs. of 4 variables:
 $ height: num 152 140 137 157 145 ...
 $ weight: num 47.8 36.5 31.9 53 41.3 ...
 $ age   : num 63 63 65 41 51 35 32 27 19 54 ...
 $ male  : int 1 0 0 1 0 1 0 1 0 1 ...
```

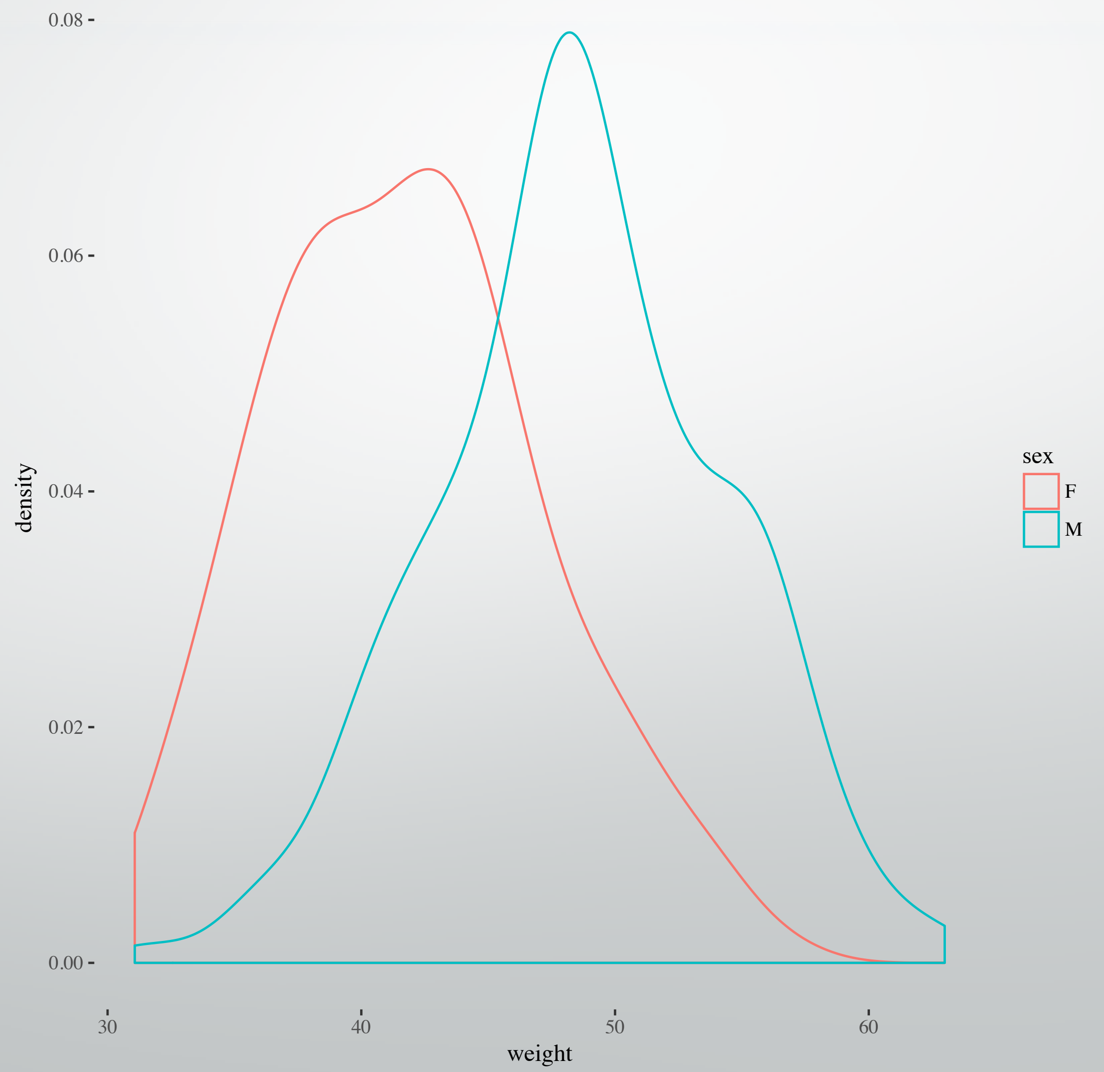


Comparison of Male vs. Female Height among !Kung San



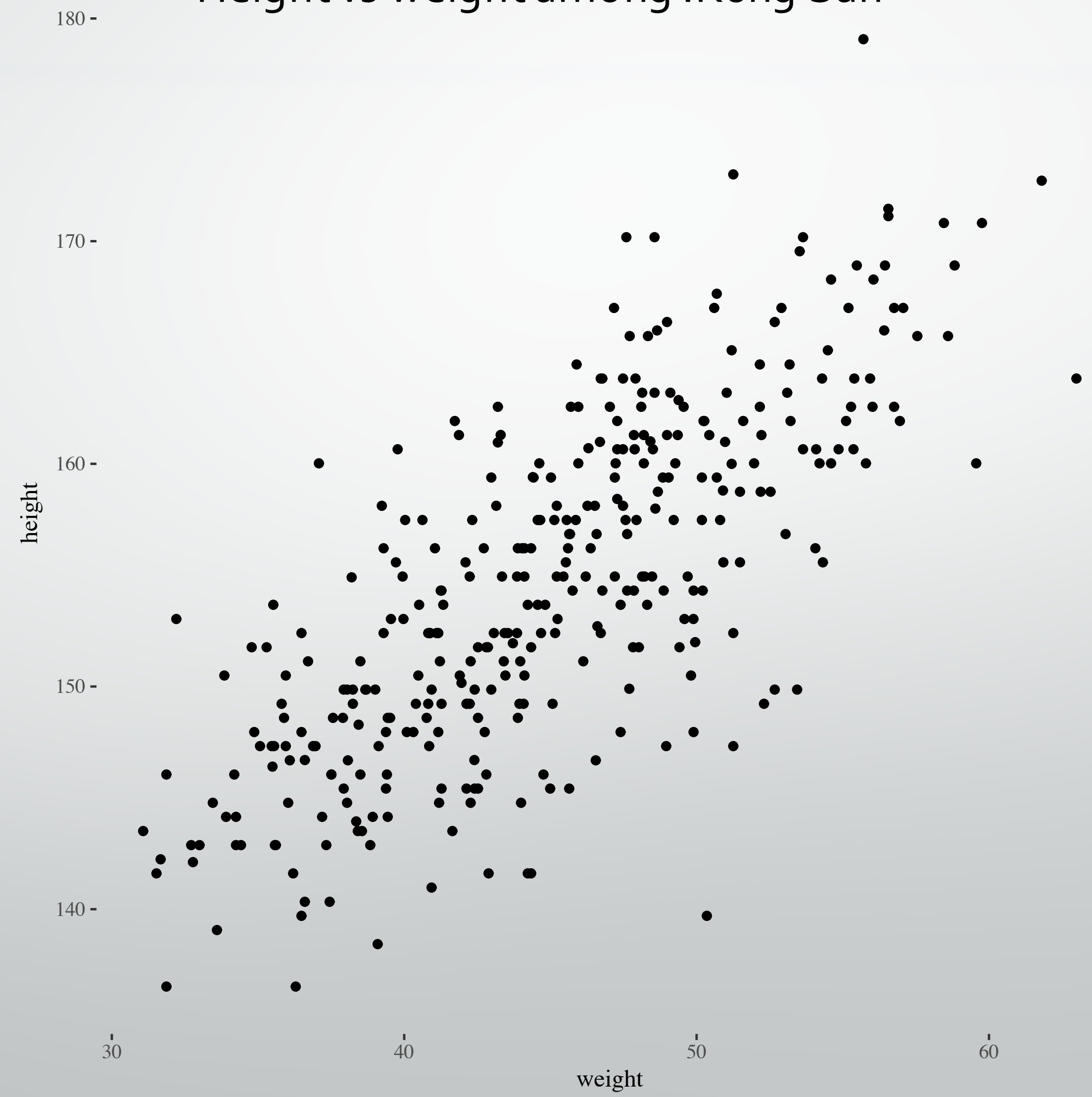
Problem | Model | Fake Data | Fit | Diagnostics | Graph fit | PPCs | Compare

Comparison of Male vs. Female Weight among !Kung San



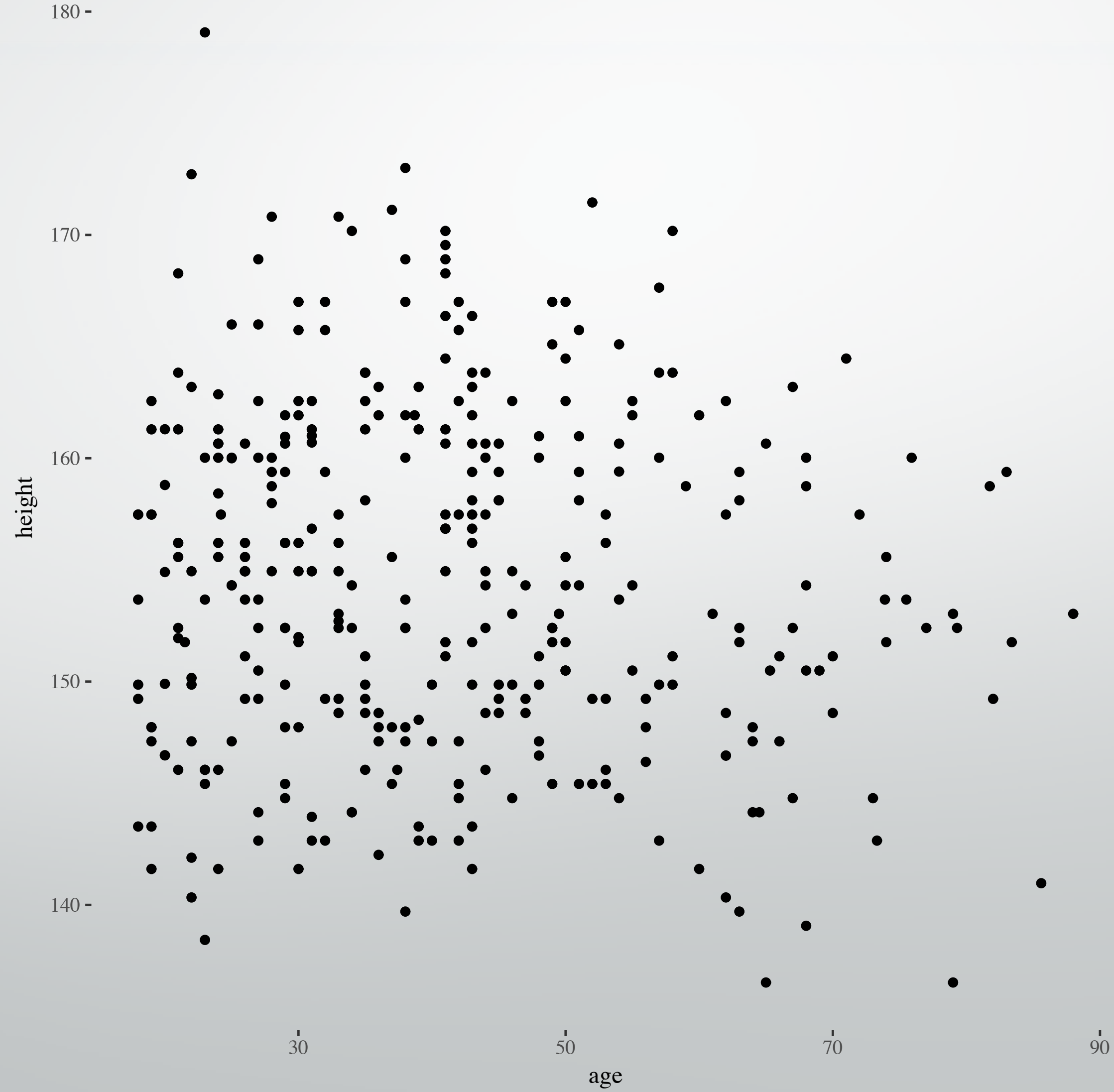
Problem | Model | Fake Data | Fit | Diagnostics | Graph fit | PPCs | Compare

Height vs weight among !Kung San



Problem | Model | Fake Data | Fit | Diagnostics | Graph fit | PPCs | Compare

Height vs age among !Kung San



Model height as a function of weight:

```
data {
  int num_people;
  vector<lower=0>[num_people] weights;
  vector<lower=0> heights[num_people];
}
```

Construct mathematical model you think generated the data:

$$\text{height} \sim \mathcal{N}(\beta * \text{weight} + \alpha, \sigma)$$

In Stan, we'd now write the parameters of this model:

```
parameters {
  real beta;
  real alpha;
  real<lower=0> sigma;
}
```

Construct mathematical model you think generated the data:

$$\text{height} \sim \mathcal{N}(\beta * \text{weight} + \alpha, \sigma)$$

And then write the likelihood:

```
model {
  heights ~ normal(beta * weights + alpha, sigma);
}
```

Problem | **Model** | Fake Data | Fit | Diagnostics | Graph fit | PPCs | Compare

$$\text{height} \sim \mathcal{N}(\beta * \text{weight} + \alpha, \sigma)$$

Think about reasonable priors for your parameters:

- Beta measures the association between weight and height, in cm/kg
- Alpha is the intercept, or average height for someone with no weight (not a particularly useful number on its own)
- Sigma is the standard deviation capturing un-modeled variation in the population

In Stan:

```
model {
  heights ~ normal(beta * weights + alpha, sigma);
  beta ~ normal(0, 10); // cm/kg
  alpha ~ normal(50, 50); // avg cm for 0 kg
  sigma ~ normal(0, 5); // variation from average
}
```

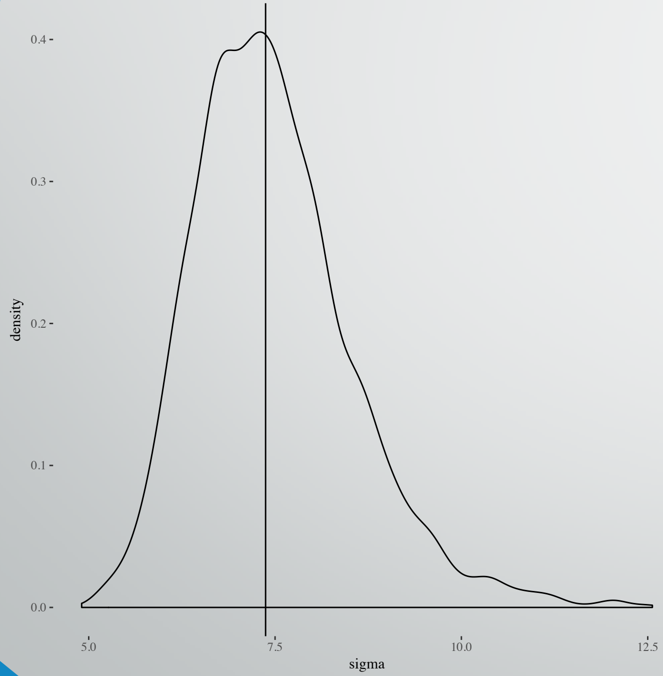
Problem | Model | **Fake Data** | Fit | Diagnostics | Graph fit | PPCs | Compare

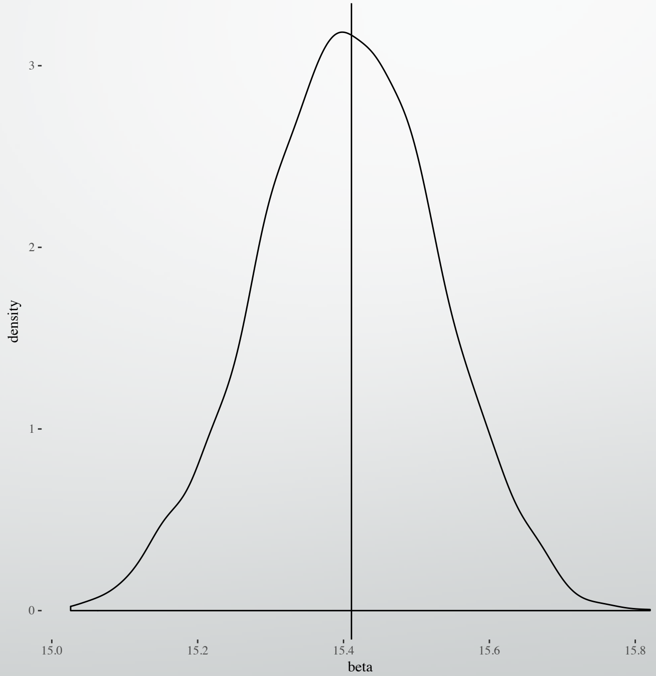
Sanity check:

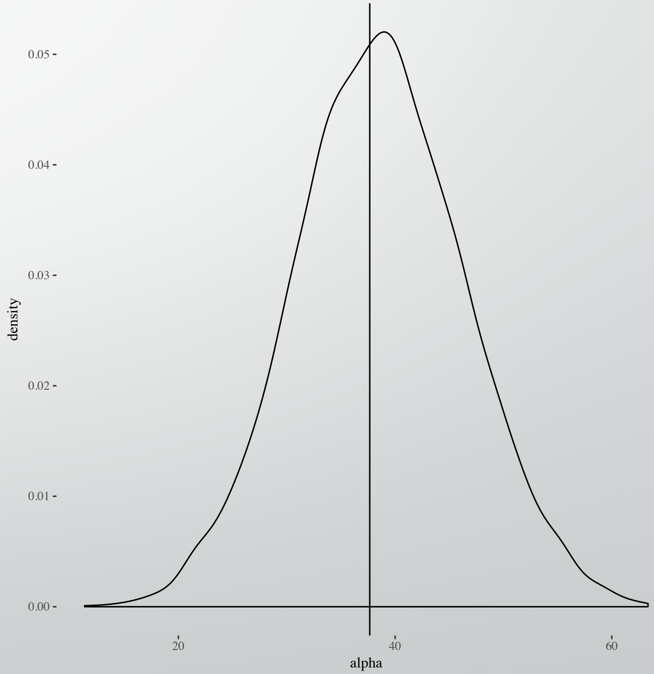
1. Draw parameter values from priors
2. Generate data based on those parameter values
3. Fit model to generated data
4. Check fit is reasonable

```
generated quantities {
  real<lower=0> heights[N];
  real beta = normal_rng(0, 10);
  real alpha = normal_rng(50, 50);
  real sigma = fabs(normal_rng(0, 5));
  for (n in 1:N)
    heights[n] = normal_rng(beta * weights[n] + alpha, sigma);
}
```

Problem | Model | **Fake Data** | Fit | Diagnostics | Graph fit | PPCs | Compare



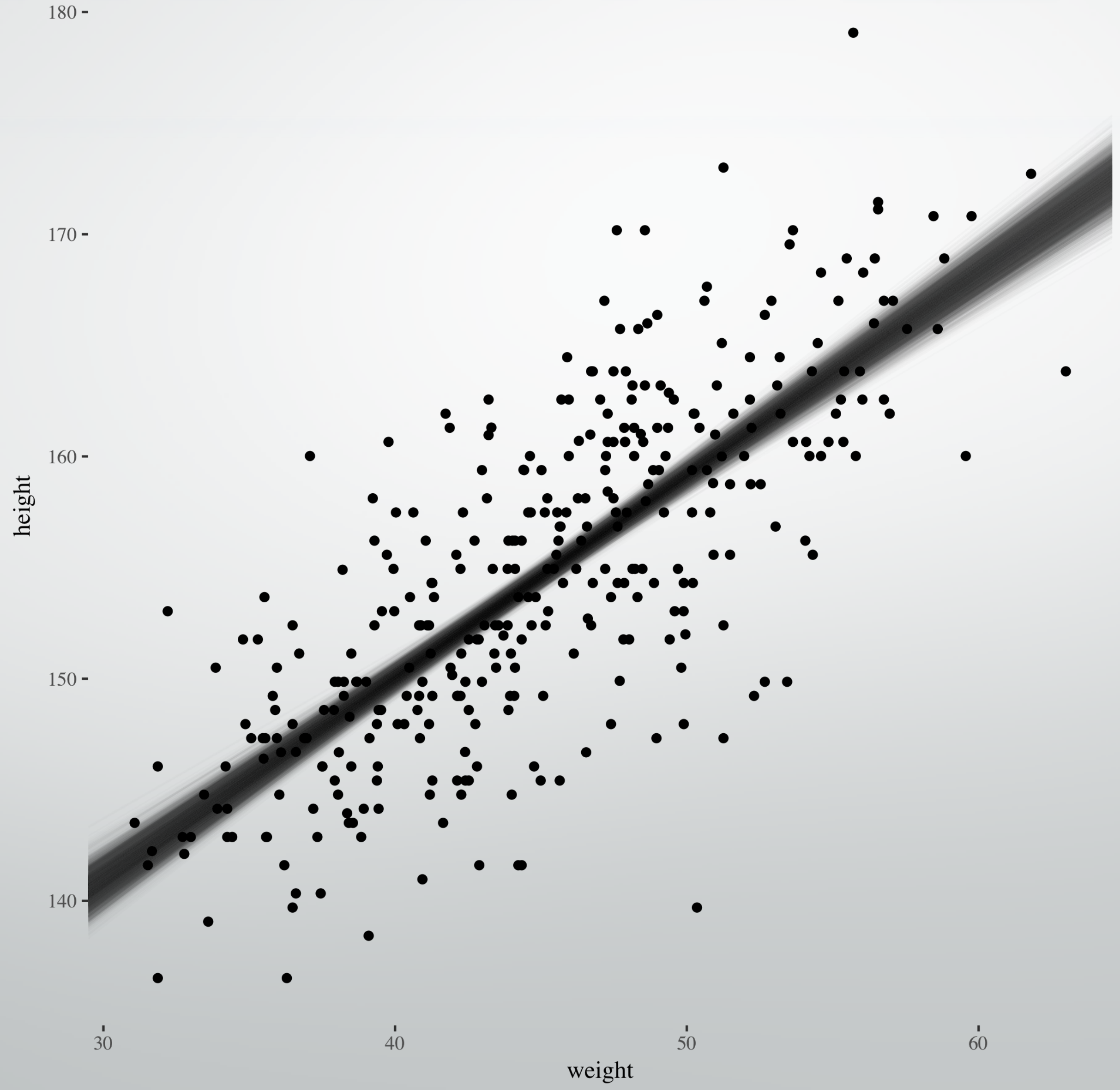




```
fit.real = sampling(heights_workflow, list(N = nrow(hdata),
  heights=hdata$height, weights=hdata$weight))
```

```
Inference for Stan model: heights_workflow.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.
       mean se_mean   sd  2.5%   25%   50%   75% 97.5% n_eff Rhat
alpha 38.60    0.20 7.69 23.66 33.37 38.53 43.73 54.10  1427    1
beta  15.41    0.00 0.12 15.16 15.32 15.41 15.49 15.64  1432    1
sigma  7.48    0.03 1.07  5.80  6.73  7.35  8.06  9.98  1248    1
```

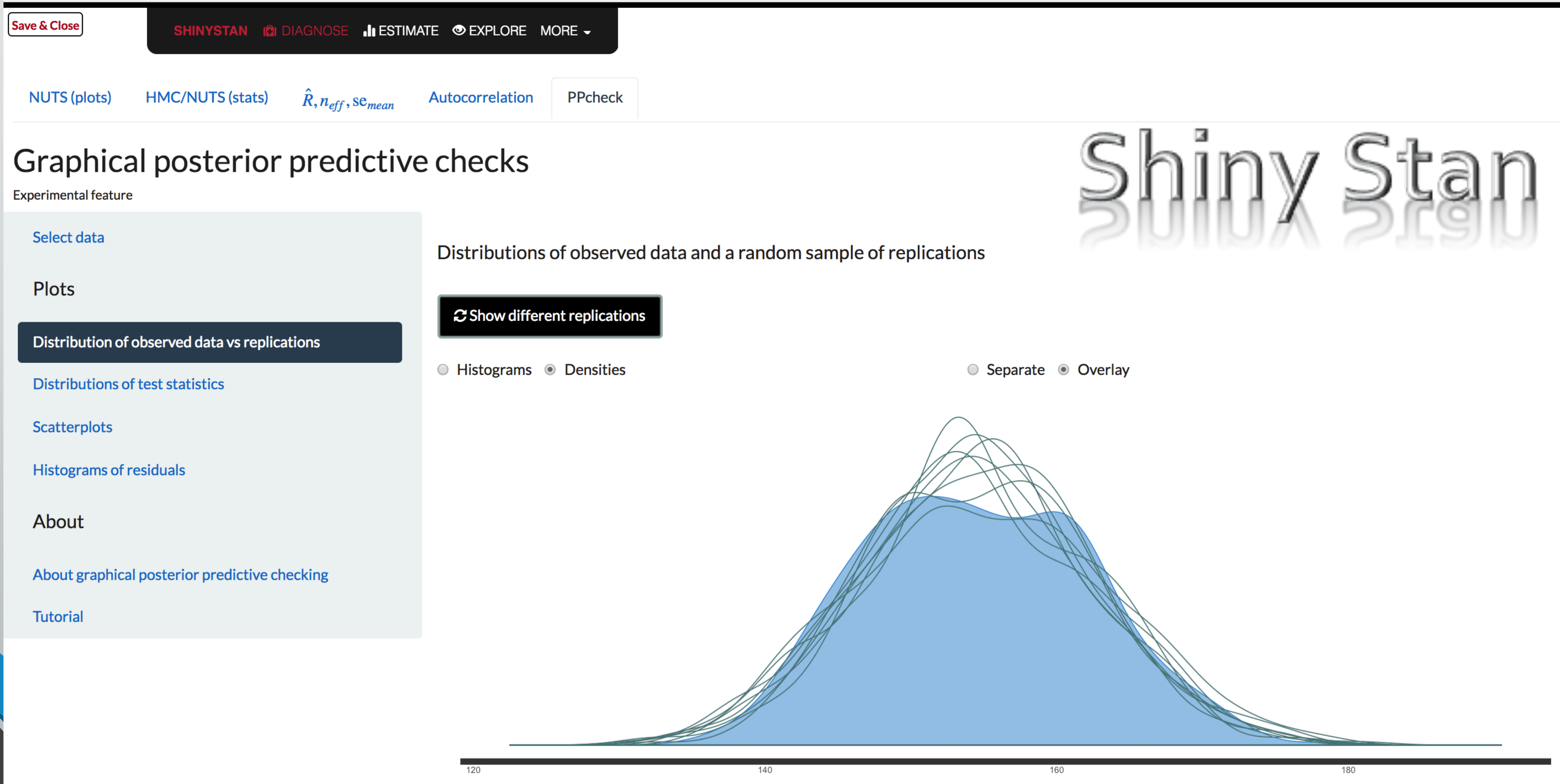
```
Warning messages:
1: There were 13 divergent transitions after warmup. Increasing
adapt_delta above 0.8 may help. See http://mc-
stan.org/misc/warnings.html#divergent-transitions-after-warmup
2: There were 70 transitions after warmup that exceeded the
maximum treedepth. Increase max_treedepth above 10. See http://mc-
stan.org/misc/warnings.html#maximum-treedepth-exceeded
3: There were 4 chains where the estimated Bayesian Fraction of
Missing Information was low. See http://mc-
stan.org/misc/warnings.html#bfmi-low
4: Examine the pairs() plot to diagnose sampling problems
```


180
170
160
150
140
height
30
40
50
60
weight

For each sample from parameter space, generate some fake measurements, and see how they match up against the real measurements.

```
generated quantities {
  real h_ppc[N];
  for (n in 1:N)
    h_ppc[n] = normal_rng(beta * weights[n] + alpha, sigma);
}
```

Problem Model Fake Data Fit Diagnostics Graph fit **PPCs** Compare



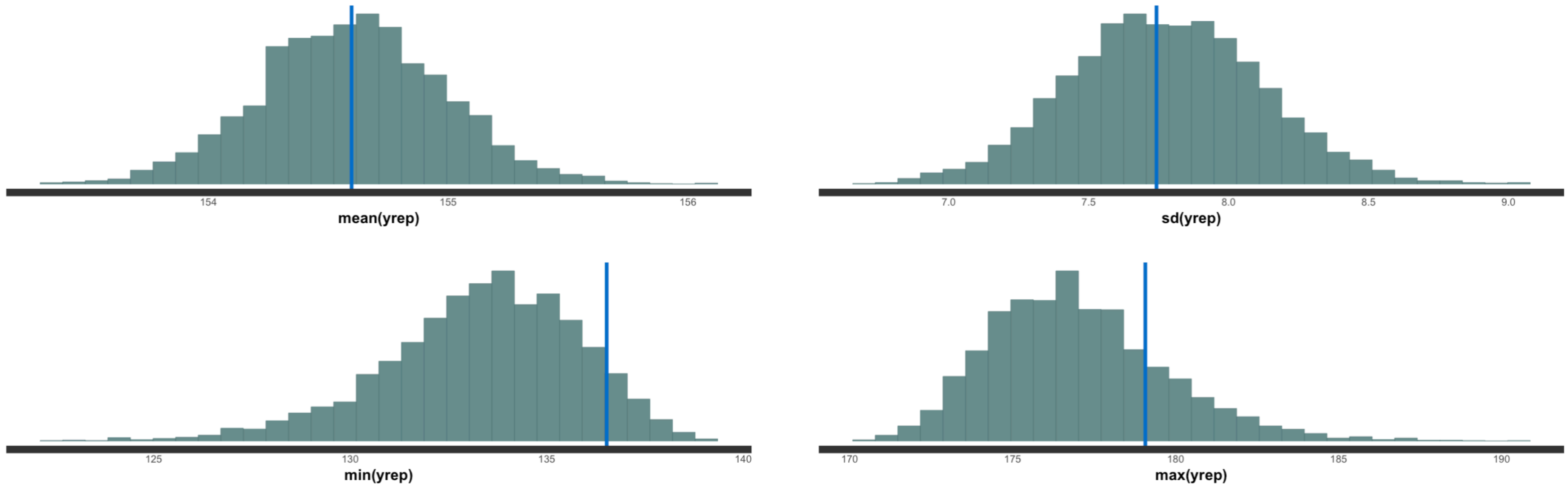
Problem | Model | Fake Data | Fit | Diagnostics | Graph fit | **PPCs** | Compare

Shiny Stan

Distributions of test statistics $T(y^{rep})$

The blue lines show $T(y)$, the value of the statistic computed from the observed data.

◉ Histograms ○ Densities



Problem | Model | Fake Data | Fit | Diagnostics | Graph fit | PPCs | **Compare**

Use these tools to compare predictive distributions and other quantities of interest among models.

Iterate!

Problem | Model | Fake Data | Fit | Diagnostics | Graph fit | PPCs | **Compare**